Analysis of LSD with One missing observation

Consider a LSD with m treatments, each treatment replicated m times. m treatments are allocated at random in m rows and m columns so that each treatment occurs once and only once in any row or any column. Thus, the total number of experimental units is m^2.

Let us assume that the yield from the plot in the i^{th} row, j^{th} column and receiving the k^{th} treatment viz, y_{ijk} is missing.

	Columns					Totals
	1	2	... j ...		m	
1						$T'_{1..}$
2						$T'_{2..}$
⋮						⋮
i			$y_{ijk} = x$			$T'_{i..} + x$
⋮						⋮
m						$T'_{m..}$
Totals	$T'_{.1.}$	$T'_{.2.}$ ...	$(T'_{.j.} + x)$	...	T	$T_{...} + x$ G

Let $y_{ijk} = x$; (say)

where

$T'_{i..}$ is the total of observations in the i^{th} row, not containing the miss. obser.

$T'_{.j.}$ is the total of observations in the j^{th} column. not con. the miss. obser.

$T'_{..k}$ is the total of observations in the k^{th} treatment not contain the miss. obser.

$T_{...}$ is the total of all (m^2-1) known observations.

$T_{i..}$ is the $(m-1)$ known obser. in the i^{th} row, containing the miss. obser.

$T_{.j.}$ is the $(m-1)$ known obser. in the j^{th} row, containing the miss. observation

$T_{..k}$ is the total of $(m-1)$ obser for the k^{th} treatment containing the miss obser

The linear mathematical model for LSD with one missing observations is

$$y_{i'j'k'} = \mu + r_{i'} + c_{j'} + t_{k'} + \epsilon_{i'j'k'} \rightarrow (1)$$

$(i' = 1,2,\ldots, i-1, i+1, \ldots, m ; j' = 1,2,\ldots, j-1, j+1, \ldots, m ; k' = 1,2,\ldots, k-1, k+1, \ldots m)$

$$y_{ijk} = \cancel{x} = \mu + r_i + c_j + t_k + \epsilon_{ijk} \rightarrow (2)$$

where

$r_{i'}(r_i)$ is the additional effect due to i'^{th} (i^{th}) row

$c_{j'}(c_j)$ is the " effect due to j'^{th} (j^{th}) Column

$t_{k'}(t_k)$ is the add. eff. due to k'^{th} (k^{th}) treatment

such that $\sum_{i=1}^{m} r_i = 0$; $\sum_{i=1}^{m} c_j = 0$; $\sum_{i=1}^{m} t_k = 0$; and

$\epsilon_{i'j'k'}(\epsilon_{ijk})$ are i.i.d $N(0, \sigma^2)$.

For the model (1), we have

$$\text{Total } S.S = \text{Raw. } S.S - C.F = \sum_{i',j',k' \in S} y_{i'j'k'}^2 + x^2 - \frac{(T_{...}+x)^2}{m^2}$$

$$S.S \text{ due to Rows } (S.S.R) = \frac{1}{m}\sum_{i'} T_{i'..}^2 + \frac{(T_{i..}+x)^2}{m} - \frac{(T_{...}+x)^2}{m^2}$$

$$S.S. \text{ due to Column } (S.S.C) = \frac{1}{m}\sum_{j'} T_{.j'.}^2 + \frac{(T_{.j.}+x)^2}{m} - \frac{(T_{...}+x)^2}{m^2}$$

$$S.S. \text{ due to Treatment } (S.S.T) = \frac{1}{m}\sum_{k'} T_{..k'}^2 + \frac{(T_{..k}+x)^2}{m} - \frac{(T_{...}+x)^2}{m^2}$$

$$\therefore ESS = T.S.S - S.S.R - S.S.C - S.S.T$$

$$= \sum_{i'j'k' \in S} y_{i'j'k'}^2 + x^2 - \frac{1}{m}\left[\sum_{i' \neq i} T_{i'..}^2 + (T_{..}+x)^2\right]$$

$$-\left[\frac{1}{m}\left\{\sum_{j' \neq j} T_{.j'.}^2 + (T_{.j.}+x)^2\right\} - \frac{(T_{...}+x)^2}{m^2}\right]$$

$$-\left[\frac{1}{m}\left\{\sum_{k' \neq k} T_{..k}^2 + \frac{(T_{..k}+x)^2}{m} - \frac{(T_{...}+x)^2}{m^2}\right]\right. \rightarrow (3)$$

$$= x^2 - \frac{1}{m}\left[(T_{i..}+x)^2 + (T_{.j.}+x)^2 + (T_{..k}+x)^2\right] +$$

$$2 \cdot \frac{(T_{...}+x)^2}{m^2}$$

$$+ \text{ Const. terms independent of } x.$$

Using the principle of least squares, we estimate x by minimising S.S.E. For S.S.E to be minimum for variation in x, we must have

$$\frac{d}{dx}(SSE) = 0$$

$$\Rightarrow 2x - \frac{2}{m}\left[(T_{i..}+x)+(T_{.j.}+x)+(T_{..k}+x)\right] + 4\frac{(T_{...}+x)}{m^2} = 0$$

$$\Rightarrow (m^2-3m+2)x = m(T_{i..}+T_{.j.}+T_{..k}) - 2T_{...}$$

$$\Rightarrow \boxed{x = \frac{m(T_{i..}+T_{.j.}+T_{..k}) - 2T_{...}}{(m-1)(m-2)}} \rightarrow (4).$$

Factorial Experiments (UNIT - 2)

The factorial expts are particularly useful in experimental situations which require the examination of the effects of varying two or more factors. In the analysis of the experimental results, the effect of each factor can be determined with the same accuracy as if only one factor had been varied at a time and the interaction effects between the factors can also be evaluated.

In facto. expt. the effect of several factors of variation are studied and investigated simultaneously, the treatments being all the combinations